

Mathematical induction is a unique and special way to prove the things, in only two steps.

Step 1. Show that it is true for $n = 1$. *Step 2.* Show that if $n = k$ is true then $n = k+1$ is also true.

For example:

Prob. By principal of mathematical induction prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133, $n \in \mathbb{N}$.

Solution. *Step 1* $P(1)$ - Show it is true for $n = 1$ $11^{1+2} + 12^{2 \cdot 1 + 1} = 11^{1+2} + 12^{2(1)+1} = 1331 + 1728 = 3059$

Yes 3059 is divisible by 133. $11^{1+2} + 12^{2(1)+1}$ is true.

Step 2 $P(k)$ - Assume it is true for $n = k$

$11^{k+2} + 12^{2(k)+1}$ is true.

(above line is an assumption only, which we will use as a fact in rest of the solution)

Now, prove that $11^{(k+1)+2} + 12^{2(k+1)+1}$ is divisible by 133. (here $n = k + 1$ now, $P(k+1)$) We have, $P(k+1)$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = 11^{k+3} + 12^{2k+3}$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = 11^{k+2} \times 11 + 12^{2k+1} \times 12^2$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = (11^{k+2} \times 11) + (12^{2k+1} \times 144)$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = (11^{k+2} \times 11) + (12^{2k+1} \times (11 + 133))$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = (11^{k+2} + 12^{2k+1}) \times 11 + (12^{2k+1} \times 133)$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = ((11^{k+2} + 12^{2k+1}) \times 11) + (12^{2k+1} \times 133)$$

Here $11^{k+2} + 12^{2k+1}$ is divisible by 133 as assumed in $n = k$, $P(1)$,

And $12^{2n+1} \times 133$ is multiple of 133 so it is divisible by 133.

So,

$$11^{(k+1)+2} + 12^{2(k+1)+1} = ((\text{divisible by } 133) \times 11) + (\text{divisible by } 133)$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = \text{divisible by } 133.$$

In this problem

If $n = n$, i.e, $P(1)$ is true then $n = n+1$, i.e, $P(n+1)$ is also true. Hence proved.

Related Posts:

1. SET
2. Relation
3. Net 34
4. prove that- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. Prove that- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. prove that - $(A \cap B) \cap (C \cap D) = (A \cap C) \cap (B \cap D)$
7. Show that- $(P \cap Q) \cap (R \cap S) = (P \cap R) \cap (Q \cap S)$
8. Binary operations
9. Algebraic structure
10. Group
11. Show that $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ is group
12. Show that $a * b = b * a$
13. if $a * c = c * a$ and $b * c = c * b$, then $(a * b) * c = c * (a * b)$
14. Undirected Graph and Incident Matrix
15. Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$, $1^3 + 2^3 + 3^3 + \dots + n^3 = [n(n+1)/2]^2$

16. Prove that $G = \{-1, 1, i, -i\}$ is a group under multiplication.
17. Hasse diagram for the “less than or equal to” relation on the set $S = \{0, 1, 2, 3, 4, 5\}$