

Q. 1 Solve the given quadratic equation

$$(x-1)^3+8=0, \quad \text{given that cube roots of unity are } 1, \omega, \omega^2.$$

Solution:

Step 1:

$$(x-1)^3+8=0$$

$$(x-1)^3=-8$$

Step 2:

$$-8=8 \cdot (-1).$$

Now, -1 can be written in exponential form as: $-1=e^{i\pi}$.

So,

$$(x-1)^3=8e^{i\pi}$$

Step 3:

The cube roots of $8e^{i\pi}$ are:

$$x-1=8^{1/3} \cdot e^{i\pi/3}, 8^{1/3} \cdot e^{i(\pi/3+2\pi/3)}, 8^{1/3} \cdot e^{i(\pi/3+4\pi/3)}$$

Since $8^{1/3}=2$, we get:

$$x-1=2e^{i(\pi/3)}, 2e^{i(\pi)}, 2e^{i(5\pi/3)}$$

Step 4: Simplify:

1. $x-1=2e^{i\pi/3}=2(\cos\pi/3+i\sin\pi/3)=2(1/2+i\sqrt{3}/2)=1+i\sqrt{3} \Rightarrow x=2+i\sqrt{3}$

2. $x-1=2e^{i\pi}=2(-1)=-2 \Rightarrow x=-1$

3. $x-1=2e^{i5\pi/3}=2(\cos5\pi/3+i\sin5\pi/3)=2(1/2-i\sqrt{3}/2)=1-i\sqrt{3} \Rightarrow x=2-i\sqrt{3}$

Answer: $x = -1, x = 2 + i\sqrt{3}, x = 2 - i\sqrt{3}$

Here formula used:

Euler's formula: $e^{i\theta}=\cos\theta+i\sin\theta$

Trigonometric Values

$$\cos\pi/3=1/2, \sin\pi/3=\sqrt{3}/2$$

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