

if $a*c = c*a$ and $b*c = c*b$, then $(a*b)*c = c*(a*b)$

Let $(A, *)$ be a semigroup. Show that for $a, b, c \in A$, if $ac = ca$ and $bc = cb$, then $(ab)c = c(a*b)$.

Solution:

Given, $a*c = c*a$
 $b*c = c*b, \forall a, b, c \in A$

To Show,
 $(a*b)*c = c*(a*b)$

Taking LHS,
 $(a*b)*c = a*(b*c)$ [Using associative law]
 $= a*(c*b)$ [$\because b*c = c*b$]
 $= (a*c)*b$ [Using associative law]
 $= (c*a)*b$ [$\because a*c = c*a$]
 $= c*(a*b)$ [Using associative law]

Hence proved.