Show that the set I of all integers (..., -4, -3, -2, -1, 0, 1, 2, 3, 4,...}.

Is a group with respect to the operation of addition of integers?

Sol.

1) Closure Property:
2+2 = 4;
2-2=0;
6+4=10’
4-6=-2;
We know that addition of two integers is also in integer.
i.e, a + b ∈ I, ∀ a, b ∈ I

2) Associative Property:
2+(4+6)=(2+4)+6;
2+(4-6)=(2-6)+4;
We know that addition of integer is an associative composition.
i.e, a+(b+c)=(a+b)+c, ∀ a, b, c ∈ I

3) Existence of Identity:
0+2=2+0;
0-2=-2+0;
Therefore there an element exist in given integer set which leaves no effect on operation.
O is an additive identity.
i.e, a+0=0+a, ∀ a ∈ I

4) Existence of Inverse:

2-2=0=-2+2;

3-3=0=-3+3;

Inverse of elements also exist in given group.

i.e, a + (-a) = 0 = (-a) + a, ∀ a ∈ I

Set 'I' have all the properties which a group have.

Hence I is a group with respect to addition.