Mathematical induction is a unique and special way to prove the things, in only two steps.

Step 1. Show that it is true for n = 1.Step 2. Show that if n = k is true then n = k+1 is also true.

## For example:

Prob. By principal of mathematical induction prove that  $11^{n+2} + 12^{2n+1}$  is divisible by 133,  $n \in \mathbb{N}$ .

Solution. Step 1 P(1)- Show it is true for  $n = 111^{n+2} + 12^{2n+1} = 11^{1+2} + 12^{2(1)+1} = 1331 + 1728 = 3059$ 

Yes 3059 is divisible by  $133.11^{1+2} + 12^{2(1)+1}$  is true.

Step 2 P(k)- Assume it is true for n = k

$$11^{k+2} + 12^{2(k)+1}$$
 is true.

(above line is an assumption only, which we will use as a fact in rest of the solution)

Now, prove that  $11^{(k+1)+2} + 12^{2(k+1)+1}$  is divisible by 133. (here n = k + 1 now, P(k+1)) We have, P(k+1)

$$11^{(k+1)+2} + 12^{2(k+1)+1} = 11^{k+3} + 12^{2k+3}$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = 11^{k+2} \times 11 + 12^{2k+1} \times 12^{2}$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = (11^{k+2} \times 11) + (12^{2k+1} \times 144)$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = (11^{k+2} \times 11) + (12^{2k+1} \times (11 + 133))$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = (11^{k+2} + 12^{2k+1}) \times 11 + (12^{2n+1} \times 133)$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = ((11^{k+2}+12^{2k+1})x \ 11) + (12^{2n+1} x \ 133)$$

Here  $11^{k+2}+12^{2k+1}$  is divisible by 133 as assumed in n=k, P(1),

And  $12^{2n+1}$  x 133 is multiple of 133 so it is divisible by 133.

So,

$$11^{(k+1)+2} + 12^{2(k+1)+1} = ((divisible by 133)x 11) + (divisible by 133)$$

$$11^{(k+1)+2} + 12^{2(k+1)+1} = \text{divisible by } 133.$$

## In this problem

If n = n, i.e, P(1) is true then n = n+1, i.e, P(n+1) is also true. Hence proved.

## Related posts:

- 1. SET
- 2. Relation
- 3. Net 34
- 4. prove that-  $AX(B \cap C) = (AXB) \cap (AXC)$
- 5. Prove that-An(BuC) = (AnB) u (AnC)
- 6. prove that  $-(A \cap B)X(C \cap D) = (AXC) \cap (BXD)$
- 7. Show that-(PnQ)X(RnS) = (PXR)n(QXS)
- 8. Binary operations
- 9. Algebraic structure
- 10. Group
- 11. Show that (..., -4, -3, -2, -1, 0, 1, 2, 3, 4,...) is group
- 12. Show that a\*b=b\*a
- 13. if a\*c = c\*a and b\*c = c\*b, then (a\*b)\*c = c\*(a\*b)
- 14. Undirected Graph and Incident Matrix
- 15. Prove the following by using the principle of mathematical induction for all  $n \in \mathbb{N}$ ,  $1^3 +$

$$2^3 + 3^3 + ... + n^3 = [n (n + 1)/2]^2$$

- 16. Prove that  $G = \{-1,1,i,-i\}$  is a group under multiplication.
- 17. Hasse diagram for the "less than or equal to" relation on the set  $S = \{0,1,2,3,4,5\}$