1. Range of Variable:

If x+y+z=6 and xy+yz+zx=9, find the range of x.

Solution:

- Formula: (x+y+z)2=x2+y2+z2+2(xy+yz+zx).
- Substitute: x+y+z = 6 and xy+yz+zx =9 then 62=x2+y2+z2+2(9)
- 36=x2+y2+z2+18
- x2+y2+z2=18.
- Now, y+z=6−x. And yz=(xy+yz+zx)−x(y+z)=9−x(6−x)=9−6x+x2.
- Quadratic in t: t2−(6−x)t+(x2−6x+9)=0. Discriminant ≥ 0: (6−x)2−4(x2−6x+9)≥0. Simplify: 36−12x+x2−4x2+24x−36≥0. −3x2+12x≥0. x(4−x)≥0. So 0≤x≤4.

Ans: Range of x is [0,4].

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2. Polynomial Relatio:

In polynomial ax4+bx3+cx2+dx+e=0, the product of roots = half the sum of product of roots taken two at a time. Find relation between a and e.

Solution:

- Product of roots = ea.
- Sum of product of roots two at a time = ca.

- Given: ea=12·ca.
- So, e=c2.

Ans: e=c2.

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3. Logarithmic Equation:

Solve log3(x2+2)−log9(x+1)=1.

Solution:

- Convert base: log9(x+1)=12log3(x+1).
- Equation: log3(x2+2)−12log3(x+1)=1.
- Multiply by 2: 2log3(x2+2)−log3(x+1)=2.
- Combine logs: log3((x2+2)2x+1)=2.
- So, (x2+2)2x+1=9.
- Expand: (x2+2)2=9(x+1). x4+4x2+4=9x+9. x4+4x2−9x−5=0.
- Factor: Try x=1: 1+4−9−5=−9 (not root). Try x=−1: 1+4+9−5=9 (not root). Use quadratic in disguise: Solve numerically. Approx roots: x≈1.5,−0.5.

Ans: x≈1.5,−0.5.

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4. Roots in A.P.

Find roots of $f(x)=x4-10x2-3x+18$, if roots are in A.P.

Solution:

- Let roots be $a-3d, a-d, a+d, a+3d$.
- Sum of roots = 0 (coefficient of x3 missing). So a=0. Roots: $-3d, -d, d, 3d$.
- Product of roots = constant term / coefficient = 18/1=18. Product = $(-3d)(-d)(d)(3d)=9d4$. So $9d4=18$. $d4=2$. $d=24$.
- Roots: $-324, -24, 24, 324$.

Ans: Roots are $-324, -24, 24, 324$.

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5. Cubic with Complex Root:

Solve $2x3-5x2+7x-20=0$ if one root is $3+i$.

Solution:

- If root is $3+i$, then $3-i$ also root (conjugate).
- Multiply: $(x-(3+i))(x-(3-i))=(x-3)2+1=x2-6x+10$.

- Divide polynomial by quadratic factor.
- Synthetic division: Divide $2x^3-5x^2+7x-20$ by $x^2-6x+10$.
- Result: Quotient = $2x-5$.
- So third root = 52.

Ans: Roots are $3+i, 3-i, 52$.

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6. Inequality Condition:

Question: If $(\mu^2+3\mu-4)x^2+(\mu+1)x<2$ holds for all real x, find the interval of μ.

Solution:

1. For inequality to hold for all x, coefficient of x^2 must be negative. So, $\mu^2+3\mu-4<0$.
2. Solve quadratic inequality: Roots of $\mu^2+3\mu-4=0 \rightarrow \mu=\frac{-3\pm\sqrt{9+16}}{2}=\frac{-3\pm5}{2}$. So, $\mu=1$ or $\mu=-4$.
3. Parabola opens upwards, so inequality <0 between roots. Interval: $(-4,1)$.

Ans: $\mu\in(-4,1)$.

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7. Quadratic Sum and Product:

Question: If sum and product of roots of $x^2-(\mu^2-4\mu+3)x+(2\mu^2-5\mu-6)=0$ are both less than 2, find possible values of μ.

Solution:

1. Sum of roots = $\mu^2-4\mu+3$. Condition: $\mu^2-4\mu+3<2$. → $\mu^2-4\mu+1<0$. Roots: $\mu=4\pm\sqrt{16-4}=4\pm\sqrt{12}=2\pm\sqrt{3}$. Interval: $(2-\sqrt{3},2+\sqrt{3})$.
2. Product of roots = $2\mu^2-5\mu-6$. Condition: $2\mu^2-5\mu-6<2$. → $2\mu^2-5\mu-8<0$. Roots: $\mu=5\pm\sqrt{25+64}=5\pm\sqrt{89}$. Interval: $(5-\sqrt{89},5+\sqrt{89})$.
3. Intersection of intervals gives valid μ.

Ans: $\mu\in(2-\sqrt{3},2+\sqrt{3})\cap(5-\sqrt{89},5+\sqrt{89})$.

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8. Exponential Inequality:

Question: If $4^x+(32)^{2x}-200>0$ for all real x, find the set of x.

Solution:

1. Rewrite: $(32)^{2x}=(18)^x$. So inequality: $4^x+18^x>200$.
2. For large x, 18^x dominates → inequality true. For small x, check boundary.
3. At $x=1$: $4+18=22<200$. Not valid. At $x=2$: $16+324=340>200$. Valid. So inequality holds for $x\geq 2$.

Ans: $x\in[2,\infty)$.

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9. Opposite Sign Roots:

Question: If roots of $x^2-(b^2+5b+2)x+b^2-3b=0$ are opposite in sign, find values of b.

Solution:

1. Roots opposite in sign → product <0. Product = b^2-3b. Condition: $b^2-3b<0$. → $b(b-3)<0$. So $0<b<3$.
2. Discriminant must be ≥0 for real roots. Discriminant = $(b^2+5b+2)^2-4(b^2-3b)$. Always positive for $b\in(0,3)$.

Ans: $b\in(0,3)$.

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