1. Range of Variable:

If x+y+z=6 and xy+yz+zx=9, find the range of x.

Solution:

- Formula: (x+y+z)2=x2+y2+z2+2(xy+yz+zx).
- Substitute: x+y+z = 6 and xy+yz+zx =9 then 62=x2+y2+z2+2(9)
- 36=x2+y2+z2+18
- x2+y2+z2=18.
- Now, y+z=6−x. And yz=(xy+yz+zx)−x(y+z)=9−x(6−x)=9−6x+x2.
- Quadratic in t: t2−(6−x)t+(x2−6x+9)=0. Discriminant ≥ 0: (6−x)2−4(x2−6x+9)≥0. Simplify: 36−12x+x2−4x2+24x−36≥0. −3x2+12x≥0. x(4−x)≥0. So 0≤x≤4.

Ans: Range of x is [0,4].

---

2. Polynomial Relatio:

In polynomial ax4+bx3+cx2+dx+e=0, the product of roots = half the sum of product of roots taken two at a time. Find relation between a and e.

Solution:

- Product of roots = ea.
- Sum of product of roots two at a time = ca.

- Given: $ea=12\cdot ca$.
- So, $e=c2$.

Ans: $e=c2$.

---

3. Logarithmic Equation:

Solve $\log3(x2+2)-\log9(x+1)=1$.

Solution:

- Convert base: $\log9(x+1)=12\log3(x+1)$.
- Equation: $\log3(x2+2)-12\log3(x+1)=1$.
- Multiply by 2: $2\log3(x2+2)-\log3(x+1)=2$.
- Combine logs: $\log3((x2+2)2x+1)=2$.
- So, $(x2+2)2x+1=9$.
- Expand: $(x2+2)2=9(x+1)$. $x4+4x2+4=9x+9$. $x4+4x2-9x-5=0$.
- Factor: Try $x=1$: $1+4-9-5=-9$ (not root). Try $x=-1$: $1+4+9-5=9$ (not root). Use quadratic in disguise: Solve numerically. Approx roots: $x\approx1.5,-0.5$.

Ans: $x\approx1.5,-0.5$.

---

4. Roots in A.P.

Find roots of f(x)=x4−10x2−3x+18, if roots are in A.P.

Solution:

- Let roots be a−3d,a−d,a+d,a+3d.
- Sum of roots = 0 (coefficient of x3 missing). So a=0. Roots: −3d,−d,d,3d.
- Product of roots = constant term / coefficient = 18/1=18. Product = (−3d)(−d)(d)(3d)=9d4. So 9d4=18. d4=2. d=24.
- Roots: −324,−24,24,324.

Ans: Roots are −324,−24,24,324.

---

5. Cubic with Complex Root:

Solve 2x3−5x2+7x−20=0 if one root is 3+i.

Solution:

- If root is 3+i, then 3−i also root (conjugate).
- Multiply: (x−(3+i))(x−(3−i))=(x−3)2+1=x2−6x+10.

- Divide polynomial by quadratic factor.
- Synthetic division: Divide 2x3−5x2+7x−20 by x2−6x+10.
- Result: Quotient = 2x−5.
- So third root = 52.

Ans: Roots are 3+i,3−i,52.

---

6. Inequality Condition:

Question: If (μ2+3μ−4)x2+(μ+1)x<2 holds for all real x, find the interval of μ.

Solution:

1. For inequality to hold for all x, coefficient of x2 must be negative. So, μ2+3μ−4<0.
2. Solve quadratic inequality: Roots of μ2+3μ−4=0 → μ=−3±9+162=−3±52. So, μ=1 or μ=−4.
3. Parabola opens upwards, so inequality <0 between roots. Interval: (−4,1).

Ans: μ∈(−4,1).

---

7. Quadratic Sum and Product:

Question: If sum and product of roots of $x2-(\mu2-4\mu+3)x+(2\mu2-5\mu-6)=0$ are both less than 2, find possible values of μ.

Solution:

1. Sum of roots = $\mu2-4\mu+3$. Condition: $\mu2-4\mu+3<2$. → $\mu2-4\mu+1<0$. Roots: $\mu=4\pm16-42=4\pm122=2\pm3$. Interval: $(2-3,2+3)$.
2. Product of roots = $2\mu2-5\mu-6$. Condition: $2\mu2-5\mu-6<2$. → $2\mu2-5\mu-8<0$. Roots: $\mu=5\pm25+644=5\pm894$. Interval: $(5-894,5+894)$.
3. Intersection of intervals gives valid μ.

Ans: $\mu\in(2-3,2+3)\cap(5-894,5+894)$.

---

8. Exponential Inequality:

Question: If $4x+(32)2x-200>0$ for all real x, find the set of x.

Solution:

1. Rewrite: $(32)2x=(18)x$. So inequality: $4x+18x>200$.
2. For large x, $18x$ dominates → inequality true. For small x, check boundary.
3. At $x=1$: $4+18=22<200$. Not valid. At $x=2$: $16+324=340>200$. Valid. So inequality holds for $x\geq2$.

Ans: $x\in[2,\infty)$.

---

9. Opposite Sign Roots:

Question: If roots of $x^2-(b^2+5b+2)x+b^2-3b=0$ are opposite in sign, find values of b.

Solution:

1. Roots opposite in sign → product <0. Product = b^2-3b. Condition: $b^2-3b<0$. → $b(b-3)<0$. So $0<b<3$.
2. Discriminant must be ≥ 0 for real roots. Discriminant = $(b^2+5b+2)^2-4(b^2-3b)$. Always positive for $b\in(0,3)$.

Ans: $b\in(0,3)$.

Related posts:

1. Class 11Mathematics Q1 (Quadratic equation)
2. Class 11 Mathematics Q2 (Quadratic equation)
3. Class 11 Mathematics Q3 (Quadratic Equation):
4. Class 11 Mathematics : Questions based on quadratic equations: Q4 – Q6
5. Maths: Multiple choice questions based on quadratic equations.
6. Class 11 Mathematics: JEE based questions