Prove the following by using the principle of mathematical induction for all n ∈ N

$1^3 + 2^3 + 3^3 + \ldots + n^3 = [n (n + 1)/2]^2$

Solution:

Let P (n) be the given statement.

i.e., P (n) : $1^3 + 2^3 + 3^3 + \ldots + n^3 = [n (n + 1)/2]^2$

For n = 1,

P (1) : $1^3 = [1 (1 + 1)/2]^2$

$1= [(1 \times 2)/2]^2$

1 = 1, which is true.

Assume that P (k) is true for some positive integer k.

1.e., P (k) : $1^3 + 2^3 + 3^3 + \ldots + k^3 = [k (k + 1)/2]^2$ ….(1)

We will now prove that P (k + 1) is also true.

Now, we have

$1^3 + 2^3 + 3^3 + \ldots + (k + 1)^3$

$= (1^3 + 2^3 + 3^3 + \ldots + k^3) + (k + 1)^3$

$= [k(k + 1)/2]^2 + (k + 1)^3$ … From (1)

$= [k^2 (k + 1)^2/4] + (k + 1)^3$

$= [k^2 (k + 1)^2 + 4(k + 1)^3]/4$

$= (k + 1)^2 [k^2 + 4(k + 1)]/4$

$= (k + 1)^2 [k^2 + 4k + 4]/4$

$= [(k + 1)^2(k + 2)^2]/4$

$= [(k + 1)(k + 2)/2]^2$

$= [(k + 1)(k + 1 + 1)/2]^2$

Thus, P (k + 1) is true, whenever P (k) is true.

Hence, from the principle of mathematical induction, the statement P (n) is true for all natural numbers i.e., $n \in N$.

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17. if a*c = c*a and b*c = c*b, then (a*b)*c = c*(a*b)