Prove the following by using the principle of mathematical induction for all n ∈ N

$1^3 + 2^3 + 3^3 + \ldots + n^3 = [n(n+1)/2]^2$

Solution:

Let P (n) be the given statement.

i.e., P (n) : $1^3 + 2^3 + 3^3 + \ldots + n^3 = [n(n+1)/2]^2$

For n = 1,

P (1) : $1^3 = [1(1+1)/2]^2$

$1 = [(1 \times 2)/2]^2$

1 = 1, which is true.

Assume that P (k) is true for some positive integer k.

1.e., P (k) : $1^3 + 2^3 + 3^3 + \ldots + k^3 = [k(k+1)/2]^2$ ....(1)

We will now prove that P (k + 1) is also true.

Now, we have

$1^3 + 2^3 + 3^3 + \ldots + (k+1)^3$

$= (1^3 + 2^3 + 3^3 + \ldots + k^3) + (k+1)^3$

$= [k (k + 1)/2]^2 + (k + 1)^3$ … From (1)

$= [k^2 (k + 1)^2/4] + (k + 1)^3$

$= [k^2 (k + 1)^2 + 4 (k + 1)^3]/4$

$= (k + 1)^2 [k^2 + 4 (k + 1)]/4$

$= (k + 1)^2 [k^2 + 4k + 4]/4$

$= [(k + 1)^2(k + 2)^2]/4$

$= [(k + 1)(k + 2)/2]^2$

$= [(k + 1)(k + 1 + 1)/2]^2$

Thus, P (k + 1) is true, whenever P (k) is true.

Hence, from the principle of mathematical induction, the statement P (n) is true for all natural numbers i.e., $n \in N$.

Related Posts:

1. Group
2. Undirected Graph and Incident Matrix
3. Prove that G={−1,1,i,−i} is a group under multiplication.
4. Hasse diagram for the “less than or equal to” relation on the set S= { 0,1,2,3,4,5}
5. SET
6. Mathematical induction

7. Relation
8. Net 34
9. prove that- AX(B∩C) = (AXB) ∩ (AXC)
10. Prove that- A∩(B∪C) = (A∩B) ∪ (A∩C)
11. prove that – (A∩B)X(C∩D) = (AXC)∩(BXD)
12. Show that- (P∩Q)X(R∩S) = (PXR)∩(QXS)
13. Binary operations
14. Algebraic structure
15. Show that (…, -4, -3, -2, -1, 0, 1, 2, 3, 4,…} is group
16. Show that a*b=b*a
17. if a*c = c*a and b*c = c*b, then (a*b)*c = c*(a*b)