Prove the following by using the principle of mathematical induction for all  $n \in \mathbb{N}$ ,  $1^3 + 2^3 + 3^3 + ... + n^3 = [n (n + 1)/2]^2$ 

Prove the following by using the principle of mathematical induction for all  $n \in N$  $1^3 + 2^3 + 3^3 + ... + n^3 = [n (n + 1)/2]^2$ 

## Solution:

Let P (n) be the given statement.

i.e., 
$$P(n): 1^3 + 2^3 + 3^3 + ... + n^3 = [n(n+1)/2]^2$$

For n = 1,

$$P(1): 1^3 = [1(1+1)/2]^2$$

$$1 = [(1 \times 2)/2]^2$$

1 = 1, which is true.

Assume that P (k) is true for some positive integer k.

1.e., 
$$P(k): 1^3 + 2^3 + 3^3 + ... + k^3 = [k(k+1)/2]^2 ....(1)$$

We will now prove that P(k + 1) is also true.

Now, we have

$$1^3 + 2^3 + 3^3 + ... + (k + 1)^3$$

$$= (1^3 + 2^3 + 3^3 + ... + k^3) + (k + 1)^3$$

EasyExamNotes.com Prove the following by using the principle of mathematical induction for all  $n \in \mathbb{N}$ ,  $1^3 + 2^3 + 3^3 + ... + n^3 = [n (n + 1)/2]^2$ 

Prove the following by using the principle of mathematical induction for all  $n \in \mathbb{N}$ ,  $1^3 + 2^3 + 3^3 + ... + n^3 = [n (n + 1)/2]^2$ 

= 
$$[k (k + 1)/2]^2 + (k + 1)^3 \dots$$
 From (1)

$$= [k^2 (k + 1)^2/4] + (k + 1)^3$$

$$= [k^2 (k + 1)^2 + 4 (k + 1)^3]/4$$

$$= (k + 1)^2 [k^2 + 4 (k + 1)]/4$$

$$= (k + 1)^2 [k^2 + 4k + 4]/4$$

$$= [(k + 1)^2(k + 2)^2]/4$$

$$= [(k + 1)(k + 2)/2]^2$$

$$= [(k + 1)(k + 1 + 1)/2]^2$$

Thus, P(k + 1) is true, whenever P(k) is true.

Hence, from the principle of mathematical induction, the statement P (n) is true for all natural numbers i.e.,  $n \in N$ .

## **Related Posts:**

- 1. Group
- 2. Undirected Graph and Incident Matrix
- 3. Prove that  $G = \{-1,1,i,-i\}$  is a group under multiplication.
- 4. Hasse diagram for the "less than or equal to" relation on the set  $S = \{0,1,2,3,4,5\}$
- 5. SET
- 6. Mathematical induction

EasyExamNotes.com Prove the following by using the principle of mathematical induction for all  $n \in \mathbb{N}$ ,  $1^3 + 2^3 + 3^3 + ... + n^3 = [n (n + 1)/2]^2$ 

Prove the following by using the principle of mathematical induction for all  $n \in \mathbb{N}$ ,  $1^3 + 2^3 + 3^3 + ... + n^3 = [n (n + 1)/2]^2$ 

- 7. Relation
- 8. Net 34
- 9. prove that- AX(BnC) = (AXB) n (AXC)
- 10. Prove that-An(BuC) = (AnB) u (AnC)
- 11. prove that  $-(A \cap B)X(C \cap D) = (AXC) \cap (BXD)$
- 12. Show that-(PnQ)X(RnS) = (PXR)n(QXS)
- 13. Binary operations
- 14. Algebraic structure
- 15. Show that (..., -4, -3, -2, -1, 0, 1, 2, 3, 4,...) is group
- 16. Show that a\*b=b\*a
- 17. if a\*c = c\*a and b\*c = c\*b, then (a\*b)\*c = c\*(a\*b)